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18CS36

Third Semester B.E. Degree Examination, Jan./Feb. 2021
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Verify that, for any three propositions p, q, r the compound proposition. $[p \to (q \to r)] \to [(p \to q) \to (p \to r)]$ is a tautology or not. **(06 Marks)**

b. Test for validity of following argument.
If Ravi goes out with friends, he will not study
If Ravi do not study, his father becomes angry
His father is not angry
———————————
∴ Ravi has not gone out with friends **(07 Marks)**

c. Give direct and indirect proof of following statement "Product of two odd integers is an odd integer". **(07 Marks)**

OR

2 a. For any three propositions p, q, r, prove that $[\sim p \wedge (\neg q \wedge r)] \vee [(q \wedge r) \vee (p \wedge r)] \Leftrightarrow r$ **(06 Marks)**

b. Check for validity of following argument,
If a triangle has two equal sides then it is isosceles. If a triangle is isoceles than it has two equal angles.
A certain triangle ABC does not have two equal angles
———————————
∴The triangle ABC does not have two usual sides **(07 Marks)**

c. Consider the following open statement on set of all real numbers as universe:
$p(x) : x \geq 0$ $q(x) : x^2 \geq 0$ $r(x) : x^2 - 3x - 4 = 0$ $s(x) : x^2 - 3 > 0$
Then find truth value of i) $\exists x\ p(x) \wedge q(x)$ ii) $\forall x,\ p(x) \to q(x)$ iii) $\forall x,\ q(x) \to s(x)$ iv) $\forall x,\ r(x) \vee s(x)$ **(07 Marks)**

Module-2

3 a. By mathematical induction prove that
$$1^2 + 3^2 + 5^2 + \ldots\ldots + (2n-1)^2 = \frac{1}{2}\, n\,(2n-1)\,(2n+1)$$ **(06 Marks)**

b. Find coefficient of i) x° in the expansion of $\left(3x^2 - \frac{2}{x}\right)^{15}$
ii) $x^{11}\, y^4$ in the expansion of $(2x^3 - 3xy^2 + z^2)^6$ **(07 Marks)**

c. A total amount of Rs.1500 is to be distributed to three students A, B, C. In how many ways distribution can be done in the multiples of Rs.100 if
i) Every students sets at least Rs.300
ii) A must get at least Rs.500, B and C must set at least Rs.400 each. **(07 Marks)**

OR

4 a. By mathematical induction prove that for any positive integer n the number $11^{n+2} + 12^{2n+1}$ is divisible by 133 **(06 Marks)**

b. How many positive integers n can be formed from the digits 3, 4, 4, 5, 5, 6, 7 if we want n to exceed 5,000,000. **(07 Marks)**

c. A certain question paper has 3 parts A, B, C with four questions in Part A, Five in B and Six in C. It is required to answer seven questions by selecting at least two from each part. In how many different ways student can answer seven questions. **(07 Marks)**

Module-3

5 a. Let A = {1, 2, 3, 4, 5, 6}, B = {6, 7, 8, 9, 10} and f be a function from A to B defined by f = {(1, 7) (2, 7), (3, 8) (4, 6) (5, 9) (6, 9)}. Then find $f^{-1}(6)$, $f^{-1}(9)$. If B_1 = {7, 8}, B_2 = {8, 9, 10) find $f^{1}(B_1)$, $f^{1}(B_2)$. **(06 Marks)**

b. Let A = {1, 2, 3, 4} and R be a relation on A defined by xRy if and only if x divides y. Then i) Write R as ordered pairs ii) Draw diagram iii) Write matrix of R. **(07 Marks)**

c. If f, g, h are functions from R to R defined by $f(x) = x^2$, $g(x) = x + 5$, $h(x) = \sqrt{x^2 + 2}$. Then verify that f o(goh) = (fog) oh **(07 Marks)**

OR

6 a. If 30 dictionaries in a library contain total 61,327 pages then prove that at least one of the dictionaries must have at least 2045 pages. **(06 Marks)**

b. For any three nonempty sets A, B, C prove that
i) $(A \cup B) \times C = (A \times C) \cup (B \times C)$
ii) $AX (B \cap C) = (A \times B) \cap (A \times C)$ **(07 Marks)**

c. Let A = {1, 2, 3, 4, 6, 8, 12} define a partial order R on A by xRy if and only if x divides y. Draw Hasse diagram of R. **(07 Marks)**

Module-4

7 a. For the integers 1, 2, …n, there are 11660 derangements where 1, 2, 3, 4, 5 appear in first five positions then find value of n. **(06 Marks)**

b. Determine number of integers between 1 and 300 which are i) divisible by exactly two of 5, 6, 8 ii) at least two of 5, 6, 8. **(07 Marks)**

c. Solve $a_n = 2(a_{n-1} - a_{n-2})$ for $n \geq 2$ given $a_6 = 1$, $a_1 = 2$ **(07 Marks)**

OR

8 a. Out of 30 students of a hostel 15 study history, 8 study economics, 6 study geography and 3 study all the three subjects. Show that 7 or more study none of the subjects. **(06 Marks)**

b. An apple, a banana, a mango, and an orange to be distributed to 4 boys B_1, B_2, B_3 and B_4. The boys B_1 and B_2 do not wish apple, B_3 does not want banana or mango B_1 refuses orange. In how many ways distribution can be made so that all of them are happy. **(07 Marks)**

c. Solve $a_n - 3a_{n-1} = 5 \times 3^n$ for $n \geq 1$ given $a_0 = 2$. **(07 Marks)**

Module-5

9 a. Show that following graphs in the Fig.Q.9(a)(i) and Fig.Q.9(a)(ii) are isomorphic

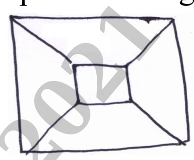

Fig.Q.9(a)(i)

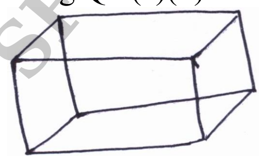

Fig.Q.9(a)(ii)

(06 Marks)

b. Define with an example to each i) Complement of a graph ii) Vertex degree iii) Rooted tree iv) Prefix code **(07 Marks)**

c. Apply merge sort to the list
-1, 7, 4, 11, 5, -8, 15, -3, -2, 6, 10, 3 **(07 Marks)**

OR

10 a. Prove that a tree with n vertices has (n – 1) edges. **(06 Marks)**

b. Determine number of vertices in following graph G:
i) G has 9 edges and all vertices have degree 3
ii) G has 10 edges with 2 vertices of degree 4 and all other have degree 3 **(07 Marks)**

c. Obtain optimal prefix code for the message ROAD IS GOOD. **(07 Marks)**

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